* Relativistic Momentum →

Consider a body of Rest mass 'm_o' moving with a velocity u. Let u_x, u_y, u_z be component of its velocity along the three Co-ordinate axis → So $u^2 = u_x^2 + u_y^2 + u_z^2$

Here $m = \frac{m_o}{\sqrt{1-\frac{u^2}{c^2}}}$

Therefore, the three components of the momentum of the body are given by $P_x = m u_x$ or $P_x = \frac{m_o u_x}{\sqrt{1-\frac{u^2}{c^2}}}$,

$P_y = m u_y$ or $P_y = \frac{m_o u_y}{\sqrt{1-u^2/c^2}}$

$P_z = m u_z$ or $P_z = \frac{m_o u_z}{\sqrt{1-u^2/c^2}}$

* Relativistic Energy →

According to Einstein mass- Energy relation $E = mc^2$

where, $m = \frac{m_o}{\sqrt{1-v^2/c^2}}$

$\therefore \; E = \frac{m_o c^2}{\sqrt{1-v^2/c^2}}$

or $E^2\left(1-\frac{v^2}{c^2}\right) = m_o^2 c^4 \Rightarrow E^2 - \frac{E^2 v^2}{c^2} = m_o^2 c^4$

$\Rightarrow E^2 = \frac{E^2 v^2}{c^2} + m_o^2 c^4 = \left[\left(\frac{E}{c^2}\right)^2 v^2\right] c^2 + m_o^2 c^4.$

Since $\frac{E}{c^2} = m$

So $E^2 = m^2 v^2 c^2 + m_o^2 c^4$

$$\boxed{E^2 = p^2c^2 + m_o^2 c^4} \quad — (1)$$

where $P = mv$ is the linear momentum of the Particle.

So $E = \pm\sqrt{p^2c^2 + m_o^2 c^4}$

This is relativistic energy relation, Thus the energy of a Particle may be Positive or negative and Predicted by Dirac.

Case → for Photon $m_o = 0$ So $E = PC$

and $P = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{c} \cdot \frac{c}{\lambda} = \frac{h}{\lambda}$

* Transformation of Momentum and Energy →

Suppose a body of Rest mass m_0 moves with velocity u in frame S, and u_x, u_y, u_z its component and

$$m = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}} \quad -①$$

For s′ frame moving with constant velocity v along x-axis have velocity of body appears to be u' and u_x', u_y', u_z' its component and

$$m' = \frac{m_0}{\sqrt{1-\frac{u'^2}{c^2}}} \quad -②$$

also $u_x' = \dfrac{u_x - v}{1-\frac{v u_x}{c^2}}$, $u_y' = \dfrac{u_y\sqrt{1-v^2/c^2}}{1-\frac{v u_x}{c^2}}$, $u_z' = \dfrac{u_z\sqrt{1-v^2/c^2}}{1-v u_x/c^2}$

The x component of the momentum of the body in frame s′ is

$$P_x' = m' u_x'$$

So $$P_x' = \frac{m_0}{\sqrt{1-\frac{u'^2}{c^2}}}\cdot\frac{u_x - v}{1-\frac{v u_x}{c^2}} \quad -③$$

eq^n ③ to express in convenient form, first evaluate the factor $\sqrt{1-\frac{u'^2}{c^2}}$ as follows →

we know $u'^2 = u_x'^2 + u_y'^2 + u_z'^2$

$$u'^2 = \frac{(u_x - v)^2 + \left(u_y\sqrt{1-\frac{v^2}{c^2}}\right)^2 + \left(u_z\sqrt{1-\frac{v^2}{c^2}}\right)^2}{\left(1-\frac{v u_x}{c^2}\right)^2}$$

$$u'^2 = \frac{(u_x - v)^2 + \left(1-\frac{v^2}{c^2}\right)(u_y^2 + u_z^2)}{\left(1-\frac{v u_x}{c^2}\right)^2} \quad -④$$

Since $u^2 = u_x^2 + u_y^2 + u_z^2$

So $u_y^2 + u_z^2 = u^2 - u_x^2$

So eq^n ④ is $$u'^2 = \frac{(u_x - v)^2 + (u^2 - u_x^2)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{v u_x}{c^2}\right)^2}$$

Now $$1-\frac{u'^2}{c^2} = 1 - \frac{1}{c^2}\,\frac{\left[(u_x - v)^2 + (u^2 - u_x^2)\left(1-\frac{v^2}{c^2}\right)\right]}{\left(1-\frac{v u_x}{c^2}\right)^2}$$

$$1-\frac{u'^2}{c^2} = 1-\frac{\left[\left(\frac{u_x}{c}-\frac{v}{c}\right)^2+\left(\frac{u^2}{c^2}-\frac{u_x^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)\right]}{\left(1-\frac{vu_x}{c^2}\right)^2}$$

$$1-\frac{u'^2}{c^2} = \frac{1+\cancel{\frac{v^2u_x^2}{c^4}}-\cancel{\frac{2vu_x}{c^2}}-\cancel{\frac{u_x^2}{c^2}}-\frac{v^2}{c^2}+\cancel{\frac{2u_xv}{c^2}}-\frac{u^2}{c^2}\left(1-\frac{v^2}{c^2}\right)+\cancel{\frac{u_x^2}{c^2}}-\cancel{\frac{v^2u_x^2}{c^4}}}{\left(1-\frac{vu_x}{c^2}\right)^2}$$

$$1-\frac{u'^2}{c^2} = \frac{\left(1-\frac{v^2}{c^2}\right)-\frac{u^2}{c^2}\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{vu_x}{c^2}\right)^2} = \frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)}{\left(1-\frac{vu_x}{c^2}\right)^2}$$

$$\therefore \sqrt{1-\frac{u'^2}{c^2}} = \frac{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}}{\left(1-\frac{vu_x}{c^2}\right)} \quad — (5)$$

This value put in equation (3) so

$$P_x' = \frac{m_0\cancel{\left(1-\frac{vu_x}{c^2}\right)}}{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}} \cdot \frac{(u_x-v)}{\cancel{\left(1-\frac{vu_x}{c^2}\right)}} = \frac{m(u_x-v)}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow P_x' = \frac{mu_x-mv}{\sqrt{1-\frac{v^2}{c^2}}}$$

Now $\boxed{P_x' = \frac{P_x-\frac{E}{c^2}v}{\sqrt{1-\frac{v^2}{c^2}}}}$

$$P_y' = m'u_y' = \frac{m_0}{\sqrt{1-\frac{u'^2}{c^2}}} \cdot \frac{u_y\sqrt{1-v^2/c^2}}{\left(1-\frac{vu_x}{c^2}\right)} = \frac{m_0\cancel{\left(1-\frac{vu_x}{c^2}\right)}}{\cancel{\sqrt{1-\frac{v^2}{c^2}}}\sqrt{1-\frac{u^2}{c^2}}} \cdot \frac{u_y\cancel{\sqrt{1-\frac{v^2}{c^2}}}}{\cancel{\left(1-\frac{vu_x}{c^2}\right)}}$$

from eqn (5)

$$P_y' = \frac{m_0u_y}{\sqrt{1-u^2/c^2}} = mu_y = P_y$$

Similarly $P_z' = P_z$.

✓ for Energy Transform equation $E' = m'c^2$

$$E' = \frac{m_0c^2}{\sqrt{1-\frac{u'^2}{c^2}}} = \frac{m_0\left(1-\frac{vu_x}{c^2}\right)c^2}{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}} \quad \text{from eq}^n \text{ (5)}$$

$$E' = \frac{mc^2\left(1-\frac{vu_x}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{mc^2-mu_xv}{\sqrt{1-\frac{v^2}{c^2}}}$$

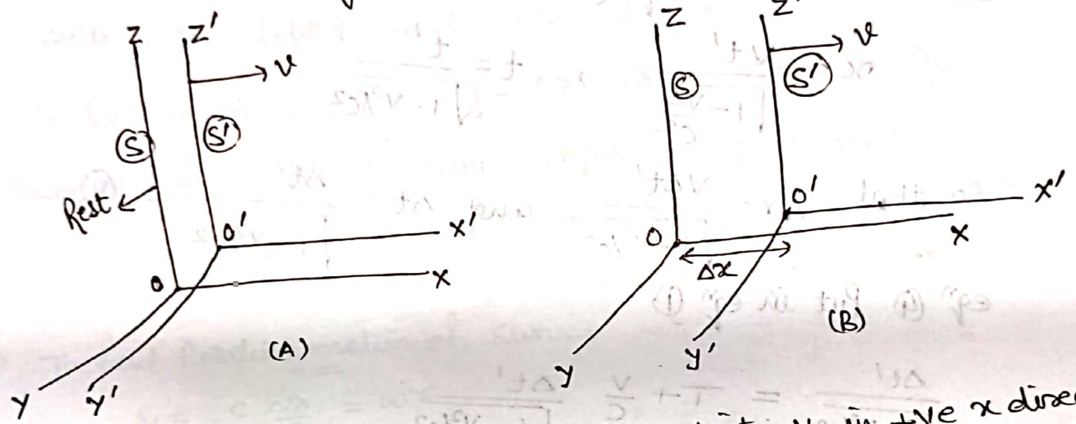
$$\boxed{E' = \frac{E - P_x v}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

The Inverse Transformation equations are →

$$P_x = \frac{P_x' + \frac{E'}{c^2} v}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad P_y = P_y', \quad P_z = P_z', \quad E = \frac{E' + P_x' v}{\sqrt{1 - v^2/c^2}}$$

* Relativistic Doppler Effect →

The change in frequency of light due to the Relative motion between Source of light and observer is called Doppler effect.



(A) (B)

Let S frame at Rest and S' moving with velocity v, in +ve x direction,

T → Time interval of two Pulses observed by observer S,

T' → " " " S'

ν and ν' freqⁿ of observer and Source at origin.

✓ Suppose the Source emits the first light Pulse at the instant when the origins of the two frames coincide for a moment see fig (A).

✓ Suppose the measurement of time started from this instant, The two Sources would record the first Pulse at the same time but the second Pulse will be recorded by the observer of frame S', After light has Traversed an extra distance Δx in frame S see fig (B).

✓ The interval $\Delta t'$ between the two Pulses recorded by the observer of frame S' is equal to time Period T' of the light Pulse, as measured by him, If this time interval Corresponds to Δt w.r.t, the observer of frame 'S' then →

Δt = Time Period of light Pulse w.r.t observer of frame S
+ time taken by light to cover distance Δx

or $\Delta t = T + \frac{\Delta x}{c}$ — ①

According to Inverse Lorentz Transformation equations

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}} \quad \text{— ②}$$

and $$t = \frac{t' + vx'/c^2}{\sqrt{1 - v^2/c^2}} \quad \text{— ③}$$

If the observer of frame S' is situated at the origin of S'
So $x' = 0$ and eqⁿ ② & ③ are

$$x = \frac{vt'}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad t = \frac{t'}{\sqrt{1 - v^2/c^2}}$$

So that $$\Delta x = \frac{v\Delta t'}{\sqrt{1 - v^2/c^2}} \quad \text{and} \quad \Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} \quad \text{— ④}$$

eqⁿ ④ Put in eqⁿ ①

$$\frac{\Delta t'}{\sqrt{1 - v^2/c^2}} = T + \frac{v}{c}\frac{\Delta t'}{\sqrt{1 - v^2/c^2}}$$

or $$T = \Delta t' \frac{(1 - v/c)}{\sqrt{1 - \frac{v^2}{c^2}}} = \Delta t' \sqrt{\frac{1 - v/c}{1 + v/c}} \quad \text{— ⑤}$$

So $$\boxed{T = T' \sqrt{\frac{1 - v/c}{1 + v/c}}} \Rightarrow \frac{T}{T'} = \sqrt{\frac{1 - v/c}{1 + v/c}}$$

or $$\frac{\nu'}{\nu} = \sqrt{\frac{1 - v/c}{1 + v/c}} \Rightarrow \boxed{\nu' = \nu \sqrt{\frac{1 - v/c}{1 + v/c}}}$$

Here $\nu' < \nu$ when observer moves away from source.

↗ If Replace v by $-v$ so $\nu' = \nu \sqrt{\frac{1 + v/c}{1 - v/c}}$

So $\nu' > \nu$ observer moves towards source.

$$\Rightarrow \frac{c}{\lambda'} = \frac{c}{\lambda} \sqrt{\frac{1 + v/c}{1 - v/c}} \Rightarrow \boxed{\lambda' = \lambda \sqrt{\frac{1 + v/c}{1 - v/c}}} \text{ or } \lambda' < \lambda$$

$$\Rightarrow \frac{\lambda'}{\lambda} = \frac{(1 + \frac{v}{c})^{1/2}}{(1 - \frac{v}{c})^{1/2}} = \left(1 + \frac{v}{2c}\right)\left(1 + \frac{v}{2c}\right)$$ By Binomial Theorem.

$$\frac{\lambda'}{\lambda} = 1 + \frac{v}{2c} + \frac{v}{2c} + \frac{v^2}{4c^2}$$ → neglected.

$$\frac{\lambda'}{\lambda} = 1 + \frac{v}{c} \Rightarrow \frac{v}{c} = \frac{\lambda'}{\lambda} - 1 = \frac{\lambda' - \lambda}{\lambda}$$

$\boxed{\Delta\lambda = \frac{v}{c}\lambda}$ → Red shift spectral line (universe expanding)

If $\boxed{\Delta\lambda = -\frac{v}{c}\lambda}$ → Violet or Blue shift spectral line.

uses →

① To find width of spectral line →.

Random motion of atom which produced Radiation have finite-width. So dopler shift $\Delta\lambda = \pm \frac{v}{c}\lambda$

So line width $= (\lambda + \Delta\lambda) - (\lambda - \Delta\lambda) = 2\Delta\lambda = \pm 2\frac{v\lambda}{c}$

Example → If velocity of atom 3 km/sec, $\lambda = 5000 A^\circ$.

$2\Delta\lambda = 10^{-11}$ mt = is very low but does see by microscope.

② To find Rotation motion of Sun. →

$$v = c\frac{\Delta\lambda}{\lambda} = \omega r.$$

So find $\omega = 2$ km/sec.

* Minskowsky Space → They gave a new concept of space-time continuum. So any event represent by four co-ordinates. Three are space (x, y, z) and one time (ct) co-ordinate.

So Lorentz's Transformation $x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{x - \frac{v}{c}(ct)}{\sqrt{1 - v^2/c^2}}$

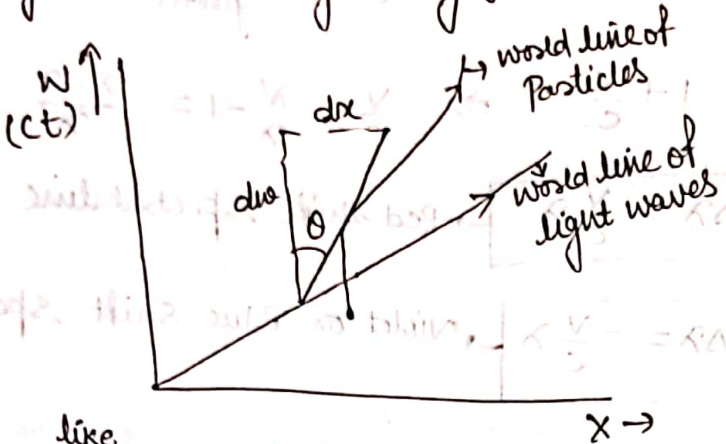
Here $w = ct$, $\frac{v}{c} = \beta$ then

$$\boxed{x' = \frac{x - \beta w}{\sqrt{1-\beta^2}}}, \quad y' = y, \quad z' = z$$

and $t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow ct' = \frac{ct - \frac{v}{c}x}{\sqrt{1 - v^2/c^2}}$

$$\boxed{w' = \frac{w - \beta x}{\sqrt{1-\beta^2}}}$$

To represent the situation geometrically. we consider space-axis only x-axis neglect y & z axis so take → (28)



(x, y, z, ct)
↓
Called minskowski world or space.

Ⓐ Space-like interval is

$(\Delta s)^2 < 0$ or (Δs) is imaginary so

$(\Delta s)^2 = c^2t^2 - x^2 - y^2 - z^2$

or $c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 < 0$

or $c^2t^2 < x^2 + y^2 + z^2$

$c < v$

Cause and effect not satisfied.

Ⓑ time like interval → $(\Delta s)^2 > 0$

So (Δs) is Real, $c > v$

Ⓒ $\Delta s = 0$ or $v = c$ for light waves.

① A point in space-time system is called the world-Point.

② The motion of a Particle in the space-time system can be Represented by a curve called world-line.

③ The Tangent to the world line at any Point gives the slope of the world line at that Point.

$$\tan\theta = \frac{dx}{dw} = \frac{dx}{d(ct)} = \frac{1}{c}\frac{dx}{dt} = \frac{u}{c}$$

where u is velocity of the Particle at that Point.

we know always $u < c$ so θ must always be acute.

④ The world line of a light wave $(u = c)$ will be a straight line making an angle of 45° with W-axis.